**Professional Development Module**

**Title:** Teaching Fractions in Grades 3 - 6

**Content and Instructional Shifts**: K-5

**Targeted Audience**: Teachers in grades 3-6

**Grade Span:** 3-6

**Description:** Instructor notes; handouts; implementation assignments – based on *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi

**Delivery time:** Session 4 of 10 three-hour sessions

The following materials were designed with the intent that the presenter(s) would be educators who have a deep understanding of the mathematical content being addressed at this level.

**Session 4 Instructor Notes:**

**Learning Goals:**

- Teachers will understand the content and instructional shifts for teaching fractions resulting from adoption of *Iowa Core Mathematics.*
- Teachers will understand the grade-specific expectations and cross grade-level learning progressions of the *Iowa Core Mathematics* fraction standards.
- Teachers will understand and implement research-based instructional strategies to build students' understanding of fractions and algebra.

**Success Criteria:**

- Teachers will classify student work for Multiple Groups multiplication and measurement division problems according to the common strategies (see *Extending Children's Mathematics* p.62).
- Teachers will describe the *Iowa Core Mathematics* fraction standards teachers can develop through the use of Multiple Group problems.
- Teachers will apply properties of operations to solve computational problems.

**Time:** 3 hours

**Materials:**

- Book *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi
- Handout "Multiple Groups Problems and Iowa Core Mathematics"
- Handout "Iowa Core Mathematics Table 3 Properties"
- Handout "Session 4 Assignment Sheet"
- Instructor Resource "Samples of Student Thinking"
- Student work collected by each participant
- Black fine point markers (2-3 per group)
- Post-it poster paper (1 per group)

**Session 4 Activity 1**
**Analyze Student Work from Implementation Assignment 3**

**Approximate Time:** 45 minutes
**Key Purpose:** To analyze student work and reflect on classroom experiences.
**Materials:**

- Book *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi
- Student work collected by each participant
- Black fine point markers (2-3 per group)
- Post-it poster paper (1 per group)

| Activity Description | Key Discussion Points |
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| **1. Small Group Work**<br>Place participants into two groups; those whose students solved the ribbon problem and those whose students solved the walking problem. Then break into grade-alike groups.<br><br>Have each participant share their students' work and how they classified the work. Also have groups discuss the following questions:<br>• Did any of your students use the number line diagram without prompting?<br>• Did you present the number line diagram during your class discussion? How did it go?<br>• What are your next steps? What problem will you give next?<br><br>Have each group create a poster showing an example of each category of strategies used by students at their grade level. | **1. Small Group Work**<br>The purpose of this activity is for teachers to:<br>• Check their categorization of student work.<br>• Discuss what they did to build student understanding of the number line diagram.<br>• Begin to plan next steps based on student understanding. Teachers make instructional decisions to further develop student understanding based on what their students understand as shown in the strategies students use. The focus should be on the mathematics teachers want students to learn. As before, discussion time is limited and this should be an ongoing discussion throughout the class. |
| **2. Whole Group Discussion**<br>Gather around each poster as an entire class and discuss the results, starting with grade 3 work. Ask participants to identify how thinking among students at different grade levels is similar and different. Compare the strategies used for the two different problems. Check if participants agree with classification of student work.<br>Discuss the following questions:<br>• What strategies did students use?<br>• Consider the direct modeling strategy. What models did students | **2. Whole Group Discussion**<br>• Teachers should find the strategies for the two different problems fall into the same categories (see *Extending Children's Mathematics* page 62).<br>• Teachers might see some progression across the grade levels. For example, more 6th grade students may use the multiplicative strategy. However, teachers will probably also see 6th grade students using direct modeling strategies. Be sure to discuss the grouping, combining, and multiplicative strategies. Also be sure to |

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| use?<br>○ Did any students use a number line? What helped them understand the number line diagram?<br>○ Did any students connect the number line to another strategy?<br>○ How did you or will you help students understand the number line?<br>• Did students use Grouping and Combining strategies? What relationships did students use? Was their thinking efficient?<br>• Did students use Multiplicative strategies? What relationships did students use? Was their thinking efficient? | discuss the number relationships students used.<br>• As students are expected to use the number line diagram for fractions, starting in 3rd grade, teachers should give students multiple opportunities to solve problems that lend themselves to this representation. The context of the problem needs to lend itself to the number line. |

**Session 4 Activity 2**
**Multiple Groups Problems and Iowa Core Mathematics**

**Approximate Time:** 45 minutes
**Key Purpose:** To recognize you can help students develop the *Iowa Core Mathematics* fraction standards through the use of Multiple Groups problems.
**Materials:**
- Book *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi
- Handout "Multiple Groups Problems and Iowa Core Mathematics"

| Activity Description | Key Discussion Points |
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| **1. Iowa Core Standards**<br>The assignment included the following question.<br>Which *Iowa Core Mathematics* standards might be developed with Multiple Groups problems? Explain your thinking.<br><br>Have participants work with a partner to complete "Multiple Groups Problems and Iowa Core Mathematics" (handout). Note this page does not repeat the standards listed on "Equal Sharing Problems and Iowa Core Mathematics" (handout from Session 2).<br><br>Discuss results as an entire class. Have participants share examples of their students' work to show evidence of student understanding of a standard. If they do not have evidence from their own student work, they may use examples of strategies from the book. | **1. Iowa Core Standards**<br>**4.NF.B.3abc Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1/*b*.**<br>**a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.**<br>**b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.***<br>**c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.**<br>• Students use addition to combine fractions and mixed numbers with like denominators.<br>• Students compose and decompose the total, representing it as the |

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| | sum of fractions and/or mixed numbers. The total may be a whole number, fraction, or mixed number.<br>• Students use the properties of operations as they combine fractions. We will discuss properties of operations later during this session.<br>**4.NFB..4abc Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.**<br>**a. Understand a fraction *a/b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).***<br>***b.* Understand a multiple of *a/b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)***<br>***c.* Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?***<br>• Students use visual fraction models to represent multiple iterations of a unit fraction.<br>• Students use visual fraction models to represent multiple iterations of a composite fraction.<br>• Students solve word problems with visual fraction models and equations. One type of Multiple Group problem involves multiplying a fraction by a whole number.<br>**5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.**<br>***a.* Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between*** |

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| | ***multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.***<br>***b.*** **Interpret division of a whole number by a unit fraction, and compute such quotients.** ***For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.***<br>***c.*** **Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.** ***For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?***<br>• A Multiple Group problem is one in which there is a whole number of groups and a fractional amount in each group where the fraction is not equal to a whole number. Some contextual problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions will fit the Multiple Groups structure and others will not.<br>• Multiple Groups partitive division problems fit the structure of a fraction divided by a whole number when the total is a fraction. For example, If 4 people share $\frac{1}{3}$ pan of brownies, how much of a pan does each person get? ($\frac{1}{3} \div 4 = ?$)<br>• Multiple Groups measurement division problems fit the structure of a whole number divided by a fraction when the total is a whole number. For example, If I have 4 pizzas, how many people can I serve $\frac{1}{5}$ pizza? ($4 \div \frac{1}{5} = ?$)<br>• Students solve Multiple Groups problems involving division using visual fraction models and equations.<br>**6.NS.A.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.** ***For example, create a story context for (2/3) ÷ (3/4) and use a visual*** |

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| | ***fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?***<br>• Some contextual problems involving division of fractions by non-zero whole numbers and division of whole numbers by fractions will fit the Multiple Groups structure and others will not.<br>• Multiple Groups partitive division problems fit the structure of a fraction divided by a whole number when the total is a fraction. For example, If 3 people share $\frac{2}{5}$ pound of chocolate, how much chocolate does each person get? ($\frac{2}{5} \div 3 = ?$)<br>• Multiple Groups measurement division problems fit the structure of a whole number divided by a fraction when the total is a whole number. For example, If I have $3\frac{3}{4}$ sub sandwiches, how many people can I serve $\frac{3}{4}$ sandwich? ($3\frac{3}{4} \div \frac{3}{4} = ?$)<br>• Students solve Multiple Groups problems involving division using visual fraction models and equations. |
| **2. Connecting Specific Multiple Group Problems to Iowa Core Mathematics Standards**<br>Have participants turn to page 65 of *Extending Children's Mathematics* and match each problem to one of the following *Iowa Core Mathematics* standard.<br>• 4.NF.B.4a (Multiply a whole number by a unit fraction.)<br>• 4.NF.B.4b (Multiply a whole number by a composite fraction.)<br>• 5.NF.B.7a (Divide a unit fraction by a whole number.)<br>• 5.NF.B.7b (Divide a whole number by a unit fraction.)<br>• 6.NS.A.1 (Divide fractions by fractions.)<br>Also have participants match the problems to a Multiple Group problem type:<br>• Multiplication<br>• Partitive division | **2. Connecting Specific Multiple Group Problems to Iowa Core Mathematics Standards**<br>You may not want to take time to classify and discuss all of the problems. The number of problems you choose will depend on the level of understanding your participants demonstrate. The purpose of this activity is to connect Multiple Groups problems to *Iowa Core Mathematics* standards and practice classifying Multiple Groups problems.<br><br>The standards do not explicitly address dividing a whole number by a composite fraction. We are classifying such problems under 6.NS.1, as this standard addresses dividing by composite fractions.<br>• Problem A - 4.NF.B.4a, multiplication<br>• Problem B - 5.NF.B.7b, measurement division ($6 \div \frac{1}{2}$) |

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| • Measurement division | • Problem C - 5.NF.B.7b, measurement division ($9 \div \frac{1}{2}$)<br>• Problem D - 5.NF.B.7b, measurement division ($3 \div \frac{1}{4}$)<br>• Problem E - 4.NF.B.4b, multiplication<br>• Problem F - 4.NF.B.4b, multiplication<br>• Problem G - 6.NS.A.1, measurement division ($3 \div \frac{3}{8}$)<br>• Problem H - 4.NF.B.4b, multiplication<br>• Problem I - 4.NF.B.4b, multiplication<br>• Problem J – 6.NS.A.1, measurement division ($4 \div \frac{2}{3}$)<br>• Problem K – 6.NS.A.1, measurement division ($12 \div \frac{3}{4}$)<br>• Problem L - 4.NF.B.4b, multiplication<br>• Problem M - 4.NF.B.4a, multiplication<br>• Problem N - 6.NS.A.1, measurement division ($7\frac{1}{2} \div \frac{3}{5}$)<br>• Problem O - 6.NS.A.1, measurement division ($10 \div \frac{2}{7}$)<br>• Problem P – 5.NF.B.7b or 6.NS.A.1 (depends on number choice), measurement division<br>• Problem Q - 5.NF.B.7b or 6.NS.A.1 (depends on number choice), measurement division<br>When the amount being shared is a fraction (see problems P and Q) the problem fits 6.NS.A.1. If the amount being shared is a unit fraction, the problem fits 5.NF.B.7a. All the problems are Multiple Groups problems. None of the problems are partitive division problems. See *Extending Children's Mathematics*, pages 29-31, for examples of partitive division problems. |
| **Session 4 Activity 3<br>Introduction to Relational Thinking** | |
| **Approximate Time: 50 minutes**<br>**Key Purpose:** To introduce relational thinking and the importance of the properties of operations when solving computational problems.<br>**Materials:**<br>• Handout "Iowa Core Mathematics Table 3 Properties"<br>• Instructor Resource "Samples of Student Thinking" | |
| **Activity Description** | **Key Discussion Points** |
| **1. Mental Math** | **1. Mental Math** |

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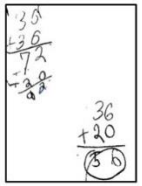
| Have participants solve the following problems mentally and share their thinking with the class. After participants share their thinking ask them which property allowed them to do what they did to solve the problem. Pass out "Iowa Core Mathematics Table 3 Properties" (handout) as a reference.<br>• 198 + 75<br>• 32 x 25<br>• 106 x 12<br>• 4 x $3\frac{1}{4}$ | The following examples show one way a participant may solve the following problems mentally. Participants will hopefully share additional ways of thinking.<br>• You might decompose 75 into 2 and 73. This allows you to add 198 and 2 for a sum of 200, then add the remaining 73 to get an answer of 273. The associative property of addition allows you to do this.<br>198 + (2 + 73) = (198 + 2) + 73<br>• You might break 32 into 8 times 4. Then the associative property of multiplication allows you to multiply 4 times 25 to get 100, and then 8 times 100 to get an answer of 800.<br>(8 x 4) x 25 = 8 x (4 x 25)<br>• You might think of 106 as 100 plus 6. Then you can determine 100 x 12 = 1200 and 6 x 12 = 72, and add the two together for answer of 1272. The distributive property of multiplication over addition allows you to do this. (100 + 6) x 12 = (100 x 12) + (6 x 12)<br>• You might use the distributive property of multiplication over addition to change the problem into two easy to multiply problems and add the results. 4 x (3 + $\frac{1}{4}$) = (4 x 3) + (4 x $\frac{1}{4}$) |
| **2. Student Thinking**<br>Show "Samples of Student Thinking" (instructor resource) and have participants explain which property or properties of operations the students used.<br>• 8 + 5 (8 plus 2 is 10, 10 and 3 more is 13.)<br>• 4 x 16 (I know 2 times 16 is 32. If I double that, I get 64.)<br>• 92 - 36<br>  <br>• 20 x 13 | **2. Student Thinking**<br>• The first student decomposed 5 into 2 + 3 and then used the associative property of addition. 8 + (2 + 3) = (8 + 2) + 3<br>• The second student broke 4 into 2 x 2 and then used the associative property of multiplication. (2 x 2) x 16 = 2 x (2 x 16)<br>• The third student used addition to solve a subtraction problem. The student also used the associative property of addition.<br>36 + ? = 92<br>(36 + 36) + ? = 92 $\rightarrow$ 72 + 20 = 92<br>36 + (36 + 20) = 92<br>36 + 56 = 92<br>• The fourth student thought of 20 times 13 as 20 times 10 plus 20 times 3. He used the distributive property of multiplication over addition. He determined the products of 20 x 10 and 20 x 3 by skip counting.<br>20 x 13 = 20(10 + 3) |

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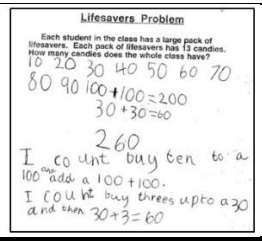
|  | 20(10 + 3) = (20 x 10) + (20 x 3)<br>• Participants will see examples of children using the properties of operations to solve fraction problems when they read chapter 4 of *Extending Children's Mathematics* after this session. |

**Session 4 Activity 4**
**Instructional Guidelines for Multiple Groups Problems**

**Approximate Time:** 30 minutes
**Key Purpose:** To plan next instructional steps and prepare for Implementation Assignment 4.
**Materials:**

- Book *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi

| Activity Description | Key Discussion Points |
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| **1. Review Instructional Guidelines**<br>Place participants in grade-alike groups. Have participants read pages 69-71 of *Extending Children's Mathematics* and then discuss the recommendations for their grade level.<br><br>Have participants read pages 67-68 of *Extending Children's Mathematics.* Ask groups to discuss the following questions:<br>• How is the approach to teaching fractions described in the opening paragraph different from traditional approaches?<br>• What is the purpose of having students solve open number sentences? | **1. Review Instructional Guidelines**<br>• The guidelines from *Extending Children's Mathematics* do not directly align with *Iowa Core Mathematics* at all grade levels. *Iowa Core Mathematics* does not explicitly address multiplication or division with whole numbers until grade 3 and multiplication with fractions until grade 4. While children are capable of solving the problems described for grades K through 2 in *Extending Children's Mathematics*, teachers of grades 3 through 6 should not expect students to have experience with the types of problems.<br>• Contextual problems help students make sense of fraction situations. Open number sentences help students apply their understanding of contextual situations to equations. This is different than the traditional approach to teaching fractions where students learn computational procedures and then apply procedural knowledge to word problems.<br>• Open number sentences help students build an understanding of the structure of fraction computation.<br>• Open number sentences allow rich discussions about number relationships. |
| **2. Plan Next Instructional Steps**<br>Ask groups to consider the following questions and determine their next steps: | **2. Plan Next Instructional Steps**<br>• Children need multiple experiences to develop a deep understanding of Iowa Core fraction standards. |

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| • How did your students do with the ribbon and/or walking problem? What other experiences do your students need with Multiple Groups problems?<br>• How did your students do with a number line? What other experiences do your students need with number lines?<br>• What problem(s) will move your students to a deeper understanding of fractions? Consider the context and number choices.<br>• How might you use open number sentences help your students develop a deeper understanding of the structure of fraction computation? What open number sentences will you pose to your students?<br>What is the next problem(s) you plan to give your students? Why did you select the problem(s)? | • Teachers should consider the grade level they teach and the level of understanding demonstrated by their students to decide what problem(s) will move their students to a deeper understanding of fractions. |

| **Session 4 Activity 5**<br>**Assignment** | |
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| **Approximate Time:** 10 minutes<br>**Materials:**<br>• Handout "Session 4 Assignment Sheet" | |
| **Activity Description** | **Key Discussion Points** |
| 1. Reading Assignment:<br>• *Extending Children's Mathematics*, Chapter 3 (pp. 65-71)<br>• *Extending Children's Mathematics*, Chapter 4 (pp. 72-91). Take notes on what you find interesting, unique, challenging, and troublesome. Be prepared to discuss your thoughts.<br><br>2. Implementation Assignment 4:<br>• Pose a Multiple Groups problem (multiplication or measurement division) to your students without providing instruction on how to solve the problem. Read the problem with your students regardless of their grade level. Read "Multiple Groups Problems" pp. 65-68, and "Instructional Guidelines for Multiple Groups Problems" pp. 69-71 for recommendations and example problems.<br>• Classify your students' work according to the strategies on p. 62 of *Extending Children's Mathematics*. | This assignment is similar to past assignments. It includes a reading assignment and implementation assignment. |

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| • Pose a couple of open number sentences to your students without providing instruction on how to solve the problem. There are good examples of open number sentences on pages 67-68. Ask select students to share how they solved the problem(s) verbally during a class discussion or by interviewing the students.<br>• Bring your students' work with you to Session 5. | |